PH 561.1 Reg. No: [| | | | | |]

St Aloysius College (Autonomous)
Mangaluru
Semester I – P.G. Examination - M. Sc. Mathematics
November - 2019

ALGEBRA I

Time: 3 Hours **Max. Marks: 70**

Answer any FIVE FULL questions from the following: **(14x5=70)**

1. a) Prove that a subset H of $\mathbb{Z}$ is a subgroup if and only if $H = a\mathbb{Z}$ for some $a \in \mathbb{Z}$.

   b) Show that, the Kernal of a group homomorphism $\phi : G \to G'$ is a normal subgroup of G.

   c) Let a, b be elements of a group G. If a has order 5 and $a^3 b = ba^3$, then prove that $ab = ba$.

(8+3+3)

2. a) Prove that, a subgroup of a cyclic group is cyclic.

   b) If H and K are normal subgroups of a group G with $H \cap K = \{e\}$ and $G = HK$, then show that G is isomorphic to $H \times K$.

   c) Let $\phi: G \to G'$ be a group homomorphism of finite group. The prove that $|G| = |\ker \phi|.|Im\, \phi|$

(6+6+2)

3. a) State converse of Lagrange's theorem. Show that it is not true by providing a counter example.

   b) Let G be a group and H be a subgroup of G. Define index of H in G. Further if $K \subseteq H$ is a subgroup, then prove that $[G:K] = [G:H]\ [H:K]$.

   c) Find all subgroups of $\mathbb{Z}_{24}$.

(6+6+2)

4. a) Let $\phi: \mathbb{R}^n \to \mathbb{R}^n$ be any function. Then prove that the following are equivalent

   i) ϕ is an isometry, which fixes the origin

   ii) ϕ preserves the dot product

   iii) ϕ is an orthogonal linear operator

   b) Let G be a finite subgroup of the group of rigid motions of the plane, which fix the origin. Then show that G is either a cyclic group of order n or G is a dihedral group D_n of order $2n$.

   c) If G acts on 'S', then show that stabilizer of $s \in S$ is a subgroup of G.

(6+6+2)

5. a) State and prove Cayley's theorem.

   b) Derive the class equation for a finite group. Determine all possible class equations for a group of order 21

(6+8)

Contd...2

6. a) State and prove first Sylow theorem.
   b) Show that every group of order 35 is cyclic.
   c) Find the order of 2-sylow subgroup of a group of order 2020.

(7+5+2)

7. a) Prove that two elements of a symmetric group S_n are conjugates if and only if they have the same cycle structure.
   b) Classify the groups of order 12.

(8+6)

8. a) Prove that, a finite integral domain is a field.
   b) Prove that, an ideal M of a ring R is maximal ideal if and only if $R/_M$ is a field.
   c) Define a prime ideal in a ring. Give an example.

(6+6+2)

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St Aloysius College (Autonomous)
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Semester I – P.G. Examination - M. Sc. Mathematics
November - 2019

LINEAR ALGEBRA I

Time: 3 Hours **Max. Marks: 70**

Answer any FIVE FULL questions from the following : **(14x5=70)**

1. a) Define the row-echelon form of a matrix. Show that any $m \times n$ matrix can be reduced to a row-echelon form by applying finitely many elementary row operations.

   b) Prove that a square matrix A is invertible if and only if $detA \neq 0$.

   c) If A and B are square matrices of same size, compute the trace of $AB - BA$.

(8+4+2)

2. a) Prove that $det\,(AB) = det\,A\,det\,B$ for square matrices A and B.

   b) Prove that $A\,(adj\,A) = (\det A)\,I_n$ for a $n \times n$ matrix A.

   c) Determine the permutation matrix associated with the permutation $p = (1\,2\,4)(3\,5) \in S_6$.

(7+5+2)

3. a) Define a basis for a vector space V over a field $\mathbb{F}$. Show that a finite set S of vectors in V which spans V contains a basis of V.

   b) If S and L are finite subsets of a vector space V such that S spans V and L is linearly independent, then prove that $|S| \geq |L|$. Deduce that any two bases of a finite dimensional vector space have the same number of elements.

   c) Determine the matrix of change of basis from $(e_1, e_1 + e_2)$ to $(e_2, e_1 - e_2)$. **(5+7+2)**

4. a) Let V an n-dimensional vector space over a field $\mathbb{F}$ and let $\mathcal{B}$ be an ordered basis of V over $\mathbb{F}$. Prove that the collection of all ordered bases of V is $\{\mathcal{B}P : P \in GL_n(\mathbb{F})\}$

   b) Determine the order of $GL_2(\mathbb{F}_p)$ where p is a prime.

   c) Find a basis for $W = \{X : AX = 0\}$ and hence find $dim\,W$,

   where $A = \begin{bmatrix} 1 & 3 & 5 & 0 \\ 1 & 4 & 0 & 2 \\ 2 & 6 & 11 & -1 \end{bmatrix}$

(7+3+4)

Contd...2

5. a) If W_1 and W_2 are subspaces of a finite dimensional vector space V over a field $\mathbb{F}$, then prove that $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

b) If W_1 and W_2 are 3-dimensional subspaces of $\mathbb{R}^5$, then is it possible to have $W_1 \cap W_2 = \{0\}$? Justify.

c) If A is an $n \times n$ matrix with entries in a field $\mathbb{F}$, show that the columns of A forms a basis of $\mathbb{F}^n$ if and only if A is invertible.

(7+2+5)

6. a) If V and W are finite dimensional vector spaces over a field $\mathbb{F}$ and $T : V \to W$ is a linear map, then prove that $\dim V = rank\, T + nullity\, T$.

b) If $T : \mathbb{R}^3 \to \mathbb{R}^2$ is given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + y \\ x - y \end{bmatrix}$, determine $Ker\, T$. Is T onto? Justify.

c) Compute the characteristic polynomial, eigenvalues and the corresponding eigenvectors for the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

(6+3+5)

7. a) Let T be a linear operator on a vector space V of dimension n over a field $\mathbb{F}$. If the characteristic polynomial of T has n distinct roots in $\mathbb{F}$, prove that there is a basis for V with respect to which the matrix of T is diagonal.

b) Prove that the following statements are equivalent for a linear operator T on a finite dimensional vector space V over a field $\mathbb{F}$.

i) $\text{Ker}\, T > 0$

ii) $Im\, T < V$

iii) If A is the matrix of T with respect to an arbitrary basis of V, then $\det A = 0$.

iv) '0' is an eigenvalue of T.

(7+7)

8. a) Define a rigid motion of $\mathbb{R}^n$. Show that a rigid motion is the composition of an orthogonal linear operator and a translation.

b) Prove that for an $n \times n$ real matrix A, $\lim_{h \to 0} \frac{e^{hA} - F}{h} = A$.

(9+5)

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St Aloysius College (Autonomous)

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Semester I - P.G. Examination - M.Sc. Mathematics

November- 2019

REAL ANALYSIS I

Time: 3 Hours

Max.Marks:70

Answer any FIVE Full questions from the following: (14x5=70)

1.a) Define the notion of supremum for a bounded subset E of an ordered set S.

b) State and prove the Archimedean property of $\mathbb{R}$.

c) Prove that if $x > 0$ and n is a positive integer, then there exists a unique $y > 0$ such that $y^n = x$.

(2+4+8)

2. a) Let A be a countable set and let B_n be the set of all n-tuples of elements of A. Prove that B_n is countable.

b) Let A be the set of all sequences of 0's and 1's. Prove that A is uncountable.

c) In a metric space, prove that any neighbourhood of a point is an open set.

d) In a metric space, prove that the countable intersection of open sets need not be open.

(5+4+3+2)

3. a) Define compact set in a metric space. Prove that any closed subset of a compact set K in a metric space X is compact.

b) Prove that every K-cell in $\mathbb{R}^k$ is compact.

(5+9)

4. a) Prove that every bounded infinite subset of $\mathbb{R}^k$ has a limit point in $\mathbb{R}^k$.

b) Show that every nonempty perfect set in $\mathbb{R}^k$ is uncountable.

c) Define a connected set in a metric space.

(5+7+2)

5 a) In a metric space prove that any convergent sequence converges to a unique point.

b) Prove that a sequence $\{x_n\}$ in $\mathbb{R}^k$ is convergent if and only if it is a Cauchy sequence.

c) Show that a monotone sequence of real numbers converges if and only if it is bounded.

(2+7+5)

6 a) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$.

b) State and prove the ratio test for convergence of series.

c) Prove that 'e' is irrational.

d) If $\sum a_n$ is a convergent series of reals, then is it necessarily absolutely convergent ? Justify.

(3+5+4+2)

Contd...2

7 a) Let f be a continuous real function on the interval $[a,b]$. If $f(a) < f(b)$ and if 'c' is a number such that $f(a) < c < f(b)$, then prove that there exists a point $x \in (a,b)$ such that $f(x) = c$.

b) Prove that any continuous mapping of a compact metric space X into a metric space Y is uniformly continuous.

c) Consider the function f defined on $\mathbb{R}$ by

$$f(x) = \begin{cases} 0 & \textit{for } x \textit{ irrationl} \\ \frac{1}{n} & \text{for } x \neq 0 \text{ rational } x = \frac{m}{n}, n > 0,\ \gcd(m,n) = 1 \\ 1 & \text{for } x = 0 \end{cases}$$

Prove that f is continuous at every irrational point and has a simple discontinuity at every rational point.

(4+6+4)

8. a) Let f be a real function defined on $[a,b]$ and differentiable at a point $x \in [a,b]$. Prove that f is continuous at x.

b) State and prove the Chain rule for differentiation.

c) State and prove the generalized mean value theorem.

(2+6+6)

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St Aloysius College (Autonomous)
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Semester I– P.G. Examination – M.Sc. Mathematics
November - 2019
GRAPH THEORY

Time: 3 hrs. Max Marks: 70

Answer any FIVE FULL questions from the following: (14x5=70)

1. a) Define the intersection number $\omega(G)$ of a graph G. Let G be a connected graph with $p > 3$ points. If G has no triangles, then show that $\omega(G) = q$, the number of liner of G.

   b) Prove that a graph is bipartite if and only if all its cycles are even. (7+7)

2. a) For any graph G with six points, prove that G or $\bar{G}$ contains a triangle.

   b) Prove that every graph is an intersection graph.

   c) Prove that in a connected graph any two longest paths have a point in common. (6+4+4)

3. a) Let G be a connected graph with at least three points. Prove that G is a block if and only if every two points of G lie on a common cycle.

   b) Prove that a cubic graph has a cutpoint if and only if it has a bridge. (8+6)

4. a) Prove that the following statements are equivalent for a (p,q) graph G.

      i) G is a tree

      ii) Every two points of G are joined by a unique path.

      iii) G is connected and $p=q+1$.

      i) G is acyclic and $p=q+1$.

   b) Define center of a graph G. Prove that every tree has a center consisting of either one point or two adjacent points. (9+5)

5. State and prove Menger's theorem. (14)

6. a) Let G be a graph with at least $2n$ points. Suppose that for any two disjoint sets V_1 and V_2 of n points each, there exist n disjoint paths joining these two sets of points. Prove that G is n- connected.

   b) Consider a family of sets $S_1, S_2, \dots, S_m$. Prove that there exists a system of distinct representatives for this family of sets if and only if the union of any k of these sets contains at least k elements, for all k from 1 to m. (5+9)

7. a) Let G be a graph having $p \geq 3$ points. If for every n, $1 \leq n \leq \frac{p-1}{2}$, the number of points of degree not exceeding n is less than n and if, for odd p, the number of points of degree at most $\frac{p-1}{2}$ does not exceed $\frac{p-1}{2}$, then prove that G is Hamiltonian. (14)

8. a) Define the point covering number α_1 and line independence number β_1 of a graph G. For any non-trivial connected (p,q) graph G, prove that $\alpha_1 + \beta_1 = p$.

   b) Prove that every planar graph is 5 –colorable. (7+7)

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St Aloysius College (Autonomous)
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Semester I – P.G. Examination - M. Sc. Mathematics
November - 2019

OPERATIONS RESEARCH

Time: 3 Hours Max. Marks: 70

Answer any FIVE FULL questions from the following: (14x5=70)

1. a) A firm produces three products. These products are processed on three different machines. The time required to manufacture one unit of each of the three products and the daily capacity of the three machines are given below. The profit per unit for product 1, 2 and 3 is ₹4, ₹3 and ₹6 respectively. Formulate the mathematical model that will maximize the daily profit.

Machine	Time per unit (min)			Machine capacity (min/day)
	Product 1	Product 2	Product 3	
M_1	2	3	2	440
M_2	4	-	3	470
M_3	2	5	-	430

b) Define feasible solution and optimal solution of a linear programming problem.

c) A company manufactures two products X and Y whose profit contributions are ₹10 and ₹20 respectively. Product X requires 5 hours on machine I, 3 hours on machine II and 2 hours on machine III. The requirement of product Y is 3 hours on machine I, 6 hours on machine II and 5 hours on machine III. The available capacities of machine I, II and III are 30, 36 and 20 hours respectively. Find the optimal product mix.

(2+2+10)

2. a) Write the standard form of the given LPP:

Maximize $Z = 3x_1 + 5x_2 + 2x_3$

Subject to $x_1 + 2x_2 - x_3 \geq -4$

$-5x_1 + 6x_2 + 7x_3 \geq 5$

$2x_1 + x_2 + 3x_3 = 10$

$x_1,\ x_2 \geq 0$ and x_3 – unrestricted.

b) Solve by trial and error method:

A firm manufactures four different machine parts M_1, M_2, M_3, M_4, made of copper and zinc. If the table below gives the amount of copper and zinc required for each machine part, their exact availability and profit earned per unit, find the optimal mix of the machine parts to maximize the profit.

	M_1 (kg)	M_2 (kg)	M_3 (kg)	M_4 (kg)	Exact availability (kg)
Copper	10	8	4	2	100
Zinc	4	6	16	3	75
Profit (Z)	12	8	14	10	

(2+12)

Contd...2

3. Write the simplex algorithm to obtain an optimal solution (if it exists) to an LPP. **(14)**

4. Write the algorithm of two phase method and use it to solve the given problem:

Maximize $Z = x_1 + x_2$

Subject to $2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

$x_1, x_2 \geq 0$ **(14)**

5. a) Write the steps to check the optimality of a given solution to a transportation problem.

b) Find the optimum solution to the following transportation problem in which the cells contain the transportation cost in rupees:

	W_1	W_2	W_3	W_4	W_5	Available
F_1	7	6	4	5	9	40
F_2	8	5	6	7	8	30
F_3	6	8	9	6	5	20
F_4	5	7	7	8	6	10
Required	30	30	15	20	5	

(6+8)

6. a) Consider the following transportation problem:

	1	2	3	Supply
A	5	1	7	10
B	6	4	6	80
C	3	2	5	15
Demand	75	20	50	

Since there is not enough supply, some of these destinations may not be satisfied. If there are penalty costs for each unsatisfied demand given by 5,3 and 2 for destination 1,2 and 3 respectively, find the optimal solution.

b) Given below is the profit matrix of a company. Find the optimal schedule which maximizes the profit:

		Sale Agency					
		1	2	3	4	5	Production capacity
Factory	A	15	17	12	11	11	140
	B	5	9	7	15	7	190
	C	14	15	16	20	10	115
	Demand	74	94	69	39	119	

(7+7)

Contd...3

7. Write the algorithm of the Hungarian method and use it to solve the given problem:

Four different jobs can be done on four different machines. The matrix below gives the cost in rupees of producing job 'i' on machine 'j'.

Machines

Job		M_1	M_2	M_3	M_4
	J_1	5	7	11	6
	J_2	8	5	9	6
	J_3	4	7	10	7
	J_4	10	4	8	3

How should the jobs be assigned to the various machines so that the total cost is minimized? Also formulate the mathematical model of the problem.

(14)

8. Reduce the following game by dominance and find the game value:

Player B

Player A		I	II	III	IV
	I	3	2	4	0
	II	3	4	2	4
	III	4	2	4	0
	IV	0	4	0	8

(14)

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Semester I – P.G. Examination – M.Sc. Mathematics

November - 2018

ALGEBRA - I

Time: 3 Hours

Max. Marks: 70

Answer any **FIVE** FULL Questions:

(5x14=70)

1. a) Let x, y, z and w be elements of a group G with $xyz = 1$, where 1 is the identity. Does it follows that $yzx = 1$? Does it follows that $yxz = 1$?

   b) Let a and b be elements of a group G. Assume 'a' has order 7 and $a^3b = ba^3$. Prove that $ab = ba$.

   c) Prove that every subgroup of $(\mathbb{Z}, +)$ is of the form $b\mathbb{Z}$ for some $b \in \mathbb{Z}$.

(3+4+7)

2. a) How many elements of order 2 does the symmetric group S_4 contain?

   b) Prove that every subgroup of cyclic group is cyclic.

   c) Let H and K be subgroups of a group G such that $K \subseteq H$. Prove that $[G:K] = [G:H]\,[H:K]$.

(3+4+7)

3. a) Let H and K be subgroups of group G

      i) If any one of H or K is normal, then prove that HK is a subgroup of G.

      ii) If H and K normal subgroup of G, then prove that HK is also a normal subgroup of G.

   b) Prove that any subgroup of index 2 is normal.

   c) Are matrices $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$, $\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$ conjugate elements of the group $GL_2(\mathbb{R})$?

(7+4+3)

4. a) Prove that the following conditions on a map $\phi: \mathbb{R}^n \to \mathbb{R}^n$ are equivalent

      i) ϕ is an isometry that fixes the origin

      ii) ϕ preserves dot product

      iii) ϕ is an orthonormal linear operator

   b) Let G be a finite subgroup of the group of rigid motions O which fixes the origin. Then prove that G is one of the following:

      i) $G = C_n$, the cyclic group of order n generated by the rotation ρ_θ where $\theta = \frac{2\pi}{n}$

      ii) $G = D_n$, the dihedral group of order $2n$, generated by two elements, the rotation ρ_θ where $\theta = \frac{2\pi}{n}$ and a reflection r about a line through the origin.

(7+7)

5. a) State and prove Cayley's theorem.

   b) Derive the class equation of a finite group.

   c) If G is a p-group then prove that G has a non-trivial centre.

(5+5+4)

Contd...2

6. a) State and prove first Sylow theorem.

b) Let p be a prime, that divides the order of a finite group G. Prove that G contains an element of order p.

c) Let G be a group of order 72. What is the order of a 2-sylow subgroup of G? What is the order of a 3-sylow subgroup of G?

(8+4+2)

7. a) Show that every group of order 35 is cyclic.

b) Show that no group of order pq and p^2q are simple where p and q are prime numbers.

(5+9)

8. a) Let $\phi: R \to R'$ be a surjective ring homomorphism with Kernel K. Prove that there is a bijective correspondence between the set of all ideals of R' and the set of all ideals of R that contains K.

b) If p is a prime number, then prove that $\mathbb{Z}_p$ is a field and $a^{p-1} \equiv 1(mod\, p)$ for all $a \in \mathbb{Z}_p, a \neq 0$.

c) Determine the units in $\mathbb{Z}/_{12\mathbb{Z}}$.

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(8+4+2)

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Semester I – P.G. Examination - M.Sc. Mathematics
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LINEAR ALGEBRA - I

Time: 3 Hours **Max. Marks: 70**

PART - A

Answer any FIVE FULL Questions. **(14x5=70)**

1. a) Define elementary matrices. Prove that elementary matrices are invertible and their inverses are also elementary matrices.

   b) Define a row-echelon matrix. Prove that every matrix can be reduced to the row-echelon form by applying elementary row operations. **(7+7)**

2. a) Let A be a $n \times n$ matrix and E be $n \times n$ elementary matrix. Prove that $det\,(EA) = detE\ detA$. Hence prove that $det\,(AB) = det\,A\ det\,B$, for any two $n \times n$ matrices A and B.

   b) Find all the solutions of the system of equations $AX = B$, where

$$A = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 3 & 0 & 0 & 4 \\ 1 & -4 & -2 & -2 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

   c) Prove that $\{(1,2), (3,4)\}$ forms a basis of $\mathbb{R}^2$ over $\mathbb{R}$. **(7+4+3)**

3. a) For a square matrix A, prove that the following are equivalent:
      i) A can be reduced to the identity matrix by a sequence of elementary row operations.
      ii) A is a product of elementary matrices.
      iii) A is invertible.
      iv) The system of homogeneous equations $AX = 0$ has only the trivial solution.

   b) Define a permutation matrix. Prove that a permutation matrix is invertible and its inverse is the transpose matrix. **(8+6)**

4. a) Let V be a finite dimensional vector space over a field F. Prove that
      i) Every linearly independent subset L of V can be extended to a basis of V.
      ii) Any finite subset S which spans V contains a basis of V.

   b) Let S and L be finite subsets of a vector space V over a field F. If S spans V and L is linearly independent over F then prove that S contains atleast as many vectors as L does. **(7+7)**

Contd...2

5. a) If W_1 and W_2 are subspaces of a finite dimensional vector space V, prove that

$$\dim(W_1 + W_2) = \dim W_1 + dimW_2 - \dim(W_1 \cap W_2).$$

b) Determine the order of $GL_n(F_p)$, where 'p' is prime.

(8+6)

6. a) Define the characteristic polynomial of a linear operator T on a finite dimensional vector space V over a field F. Prove that the eigen values of T are the roots of the characteristic polynomial of T which lie in F.

b) Prove that for every complex $n \times n$ matrix A there is a matrix $P \in GL_n(\mathbb{C})$ such that PAP^{-1} is upper triangular.

(7+7)

7. a) For a real $n \times n$ matrix A, show that the following conditions are equivalent:

i) A is orthogonal

ii) Multiplication by A preserves dot product

iii) The columns of A are mutually orthogonal unit vectors in $\mathbb{R}^n$.

b) Find the complex eigen values of the rotation matrix $\begin{bmatrix} \cos\theta & -sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

c) Does $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ represent a rotation about the origin in $\mathbb{R}^3$? Why?

(7+4+3)

8. a) Define the exponential of an $n \times n$ real or complex matrix A. Prove that the series defining e^A converges absolutely for all complex matrices A.

b) Solve the equation $\frac{dx}{dt} = Ax$, where $A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$.

c) Compute e^A, where $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(7+5+2)

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REAL ANALYSIS I

Time: 3 Hours

Max.Marks:70

Answer any FIVE full questions.

1.a) If $x \in \mathbb{R}$, $y \in \mathbb{R}$, $x > 0$ then prove that there exists a positive integer 'n' such that $nx > y$ and hence show that there exists a rational number between any two real numbers.

b) State and prove Cauchy-Schwarz inequality. **(9+5)**

2. a) Define a countable set. Prove that every infinite subset of a countable set is countable.

b) Let A be a countable set and let B_n be the set of all n-tuples of elements of A. Prove that B_n is countable.

c) In a metric space prove that any neighbourhood of a point is an open set. **(5+6+3)**

3. a) Let A be a subset of a metric space X. Prove that A is open in X if and only if is complement A^c is closed in X.

b) Prove that every closed subset of compact set in a metric space X is also compact.

c) If $\{I_n\}$ is a sequence of non empty closed intervals in $\mathbb{R}$ such that $I_{n+1} \subseteq I_n \ \forall \ n$, then prove that $\bigcap_{n=1}^{\infty} I_n \neq \phi$. **(4+5+5)**

4. a) Prove that every $k-cell$ in $\mathbb{R}^k$ is compact.

b) If $A \subseteq \mathbb{R}^k$ is closed and bounded then show that A is compact.

c) *Define a connected set in a metric space.* **(8+4+2)**

5 a) *In a metric space prove that any convergent sequence converges to a unique point.*

b) If X is a compact metric space and if $\{p_n\}$ is a Cauchy sequence in X then show that $\{p_n\}$ converges in X.

c) Show that a monotone sequence of real numbers converges if and only if it is bounded. **(2+7+5)**

6 a) If $|a_n| \leq c_n$ for $n \geq N_o$, where N_o is some fixed integer and if $\sum c_n$ converges then prove that $\sum a_n$ converges.

b) Suppose $a_1 \geq a_2 \geq ... \geq 0$. Then prove that the series $\sum_{n=1}^{\infty} a_n$ converges if and only if the series $\sum_{k=0}^{\infty} 2^k a_{2^k} = a_1 + 2a_2 + 4a_4 + 8a_8 + ...$ converges.

c) Prove that 'e' is irrational. **(4+6+4)**

Contd...2

7 a) Prove that a mapping f of a metric space X into a metric space Y is continuous if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

b) Suppose that f is a continuous real function on a compact metric space X, and M and m are the supremum and infimum of $f(X)$. Prove that there exist points $p, q \in X$ such that $f(p) = M$ and $f(q) = m$.

(7+7)

8. a) State and prove the generalized mean value theorem.

b) Suppose $f: \mathbb{R} \to \mathbb{R}$ satisfies the inequality $|f(x) - f(y)| \le (x-y)^2, \forall\ x, y \in \mathbb{R}$. Prove that f is constant.

c) Examine the differentiability of the function f at $x = 0$ for the function

$$f(x) = \begin{cases} x^2 \sin\dfrac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

(7+3+4)

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PS 564.1

Reg. No: | | | | | | |

St Aloysius College (Autonomous)

Mangaluru

Semester I – P.G. Examination - M. Sc. Mathematics

November - 2018

GRAPH THEORY

Time: 3 Hours Max. Marks: 70

Answer any **FIVE FULL** questions from the following (14x5=70)

1. a) Show that number of points of odd degree in any graph G is even.

   b) If G is a bipartite graph, then show that all its cycles are even.

   c) Show that the maximum number of lines among all p - point graphs with no triangles is $\left[\frac{p^2}{4}\right]$, where p is even. **(3+3+8)**

2. a) Define the intersection graph on a set S. Show that every graph is an intersection graph.

   b) Prove that for any graph G with 6 points, either G or $\overline{G}$ contains a triangle.

   c) Define the notions:

      i) invariant of a graph

      ii) component of a graph

      iii) diameter of a graph **(5+6+3)**

3. a) Show that a cubic graph has a bridge if and only if it has a cut point.

   b) Prove or disprove: A connected graph G with $p \geq 3$ is a block if and only if given any two points and a line, there is a path joining the points which does not contain the line.

   c) Prove that, if v is a cut point of a graph G then v is not a cut point of the complement $\overline{G}$. **(8+4+2)**

4. For a (p,q) graph G, prove that the following are equivalent:

   i) G is a tree

   ii) Every two points of G are joined by a unique path

   iii) G is connected and $p = q + 1$

   iv) G is acyclic and $p = q + 1$. **(14)**

5. a) Define the point connectivity $\kappa(G)$ and the line connectivity $\lambda(G)$ of a graph G. Prove that $\kappa(G) \leq \lambda(G)$.

   b) Prove that a graph G with atleast n points is n connected if and only if for any two disjoint sets V_1 and V_2 of n points each, there exist n disjoint paths joining these two sets of points. **(5+9)**

Contd...2

6. Prove that the minimum number of points separating two non-adjacent points s and t in a connected graph G is the maximum number of disjoint $s-t$ paths. (14)

7. a) Define an Eulerian and a Hamiltonian graph. Give an example of a graph which is Eulerian but not Hamiltonion.

   b) Prove that the following are equivalent for a connected graph G:

   i) G is Eulerian

   ii) every point of G is of even degree

   iii) the set of lines of G can be partitioned into cycles.

   c) Define a plane map. For a plane map G with p vertices, q edges and r faces, prove that $p-q+r=2$. (3+6+5)

8. a) If G is a (p,q) plane map in which every face is an n-cycle, then prove that $q=\frac{n(p-2)}{n-2}$.

   b) When do we say that a graph is n-colorable? Prove that, every planar graph is 5-colorable. (3+11)

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PS 565.1

Reg. No: | | | | | | |

St Aloysius College (Autonomous)

Mangaluru

Semester I – P.G. Examination - M. Sc. Mathematics

November - 2018

FLUID MECHANICS

Time: 3 Hours Max. Marks: 70

Answer any **FIVE FULL** questions. (14x5=70)

1. a) Obtain Euler's equation of motion in Cartesian form.

   b) Show that the rate of increase of energy in the system is equal to the rate at which work is done on the system.

(6+8)

2. a) State and Prove Bernoulli's theorem in standard form.

   b) A long pipe is of length 'l' and has slowly tapering cross section. It is inclined at angle 'α' to the horizontal and water flows steadily and irrotationally through it from the upper end to lower end in the absence of body forces. The section at the upper end has twice the radius of the lower end. At the lower end water is maintained at atmospheric pressure P_a, where as pressure at upper end is twice atmospheric. Find the exit speed of water at lower end.

   c) Show that if a fluid flows steadily and irrotationally, then pressure is maximum at stagnation point.

(6+6+2)

3. a) State and prove Kelvin's minimum energy theorem.

   b) State and prove Kelvin's circulation theorem. Also show that a motion of a fluid is circulation preserving if and only if

$$\frac{\partial \Omega}{\partial t} + \nabla \times (\Omega \times q) = 0.$$

(5+9)

4. a) If $\emptyset$ and ψ are the velocity potential and stream function of a two dimensional flow, show that they satisfy the Laplace equation and lines of constant $\emptyset$ and ψ intersects orthogonally.

   b) Obtain the complex potential due to a doublet and hence discuss the flow due to a doublet of unit length and of unit strength.

(6+8)

5. a) State and prove Milne-Thomson circle theorem.

   b) Find the stream lines for a source and sink of equal strength placed at $\left(\pm\frac{a}{2}, 0\right)$ within a fixed cylinder boundary $x^2 + y^2 = r^2$.

(7+7)

Contd...2

6. a) State and prove the theorem of Blasius.

b) Verify that $w = iK\log\left(\frac{z-ia}{z+ia}\right)$ is the complex potential of a steady flow of liquid about a circular cylinder, the plane $y=0$ being a rigid boundary. Find the force exerted by the liquid on unit length of the cylinder.

(7+7)

7. a) Derive Navier-Stoke's equation of motion of viscous fluid.

b) Obtain the velocity distribution for the flow between two rotating cylinders.

(8+6)

8. a) What is Plane- Poiseuille flow? Obtain the velocity distribution, average velocity, maximum velocity and skin friction at the boundaries.

b) Describe the diffusion of vorticity.

(9+5)

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PH 561.1 **Reg. No:** [| | | | |]

St Aloysius College (Autonomous)

Mangaluru

Semester I – P.G. Examination - M. Sc. Mathematics

November - 2017

ALGEBRA I

Time: 3 Hours **Max. Marks: 70**

Answer any FIVE FULL questions from the following **(14x5=70)**

1. a) Prove that H is a subgroup of $\mathbb{Z}$ if and only if $H = n\mathbb{Z}$ for some $n \in \mathbb{Z}$.

   b) If $G = \langle a \rangle$ is a finite cyclic group of order n, then show that G has $\phi(n)$ generators.

   c) Let G be a group of even order. Show that there is an element a in G other than identity e such that $a^2 = e$.

(6+6+2)

2. a) State and prove first isomorphism theorem on groups.

   b) Let H and K be subgroups of a group G. Then prove that

   i) HK is a subgroup of G if and only if $HK = KH$

   ii) if both H and K are normal subgroups of G then HK is also a normal subgroup of G.

   iii) If H is a normal subgroup of G then HK is a subgroup of G.

(5+9)

3. a) If G is a finite subgroup of the group of rigid motions of the plane, show that there is a point in the plane which is fixed by every element of G.

   b) Define isometry of $\mathbb{R}^2$. Prove that every isometry of $\mathbb{R}^2$ is a translation, a rotation or a reflection.

(7+7)

4. a) State and prove first Sylow theorem.

   b) If p and q are prime numbers, then prove that no group of order pq is simple.

(9+5)

5. a) Prove that two elements in S_n are conjugates if and only if they have the same cyclic structure.

   b) Obtain class equation of symmetric group S_4.

   c) Show that S_n is generated by transpositions.

(8+2+4)

Contd...2

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6. a) If H and K are subgroups of group G, show that $[H : H \cap K] \le [G : K]$.

   b) If p is a prime which divides the order of a finite group G, then prove that G contains an element of order p.

(7+7)

7. a) Define an integral domain. Prove that every integral domain can be embedded in a field.

   b) If $R \neq \{0\}$ is a commutative ring with identity having only two ideals $\{0\}$ and itself, then prove that R is field.

(10+4)

8. a) If F is a field then prove that the polynomial ring $F[x]$ is a principal ideal domain.

   b) State and prove the correspondence theorem on ring homomorphism.

(5+9)

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562.1 Reg. No:

St Aloysius College (Autonomous)

Mangaluru

Semester I – P.G. Examination - M. Sc. Mathematics

November - 2017

LINEAR ALGEBRA I

e: 3 Hours Max. Marks: 70

Answer any FIVE FULL questions from the following (14x5=70)

1. a) Define elementary matrices. Prove that elementary matrices are invertible and their inverses are also elementary.

   b) Prove that any non zero matrix can be reduced to row echelon form by applying finitely many elementary row operations.

(6+8)

2. a) If A and B are two $n\times n$ matrices, prove that $\det(AB)=\det A.\det B$. Is $\det(A+B)=\det A+\det B$? Justify.

   b) Prove that an $n\times n$ matrix A is invertible if and only if $\det(A)$ is nonzero.

   c) If $p=(1\ \ 3\ \ 5\ \ 6\ \ 2)\in S_6$, determine the permutation matrix associated with p.

(8+4+2)

3. a) If A is an $n\times n$ real matrix, prove that $(adj\ A)A=(\det A)I_n$.

   b) Let A be an $n\times n$ matrix with integer entries. Prove that A^{-1} has integer entries if and only if $\det A=\pm 1$.

   c) Solve the system of equations:

$$x+2z+w=5$$
$$x+y+5z+2w=7$$
$$x+2y+8z+4w=12$$

(6+4+4)

4. a) Let V be a finite dimensional vector space over a field F. Prove the following:

   i) any finite set of vectors which spans V contains a basis of V.

   ii) any finite linearly independent set of vectors in V can be extended to a basis of V.

   b) Let W be a subspace of a finite dimensional vector space V over a field F. Prove that W is finite dimensional and $\dim W \le \dim V$

   c) Let P_5 be the space of all polynomials of degree ≤ 5 with real coefficients. Check whether the vectors $v_1=1+x^2, v_2=1+x-x^2, v_3=2-x+x^2$ are linearly independent over R.

(6+4+4)

Contd...2

5. a) If W_1 and W_2 are subspaces of a finite dimensional vector space V over a field F, prove that $\dim(W_1+W_2)=\dim W_1+\dim W_2-\dim(W_1\cap W_2)$. If the sum of the dimensions of two subspaces of an $n-$dimensional vector space exceeds n, prove that the subspaces have a non-zero vector in common.

b) If W is a subspace of a finite dimensional vector space V, prove that there is a subspace W' of V such that $V=W\oplus W'$. If $W=Ker\,A$, where $A=\begin{bmatrix}1 & 2 & 1\\ 1 & -1 & 3\end{bmatrix}$, find W' such that $W\oplus W'=\mathbb{R}^3$.

(8+6)

6. a) State and prove the rank-nullity theorem.

b) If $T:\mathbb{R}^2\to\mathbb{R}^3$ is a linear transformation given by $T\left((x,y)^t\right)=(x,\ x+y,\ y)^t$, find the kernel of T. Is T one-one? Is T onto?

c) Diagonalize the matrix $\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$.

(6+3+5)

7. a) Let $T:V\to W$ be a linear map of the vector spaces V, W over a field F, of dimensions n,m respectively. Prove that there exist bases $\mathcal{B}$, C of V,W respectively, such that the matrix of T with respect to $\mathcal{B}$ and C is of the form $\begin{bmatrix}I_r & 0\\ 0 & 0\end{bmatrix}$ where $r=rank\,T$.

b) Prove that the following statements are equivalent for a linear operator T on a finite dimensional vector space V over a field F.

i) $\ker T \supsetneq 0$

ii) $imT \subsetneq V$

iii) If A is the matrix of T with respect to an arbitrary basis of V, then $\det A=0$.

iv) 0 is an eigen value of T.

8. a) Prove that an $n\times n$ real matrix A is orthogonal if and only if $(AX\,.\,AY)=(X\,.\,Y)$ for all $X,\ Y\in\mathbb{R}^n$.

(7+7)

b) Define a rigid motion of $\mathbb{R}^n$. Prove that a rigid motion which fixes the origin is a linear operator on $\mathbb{R}^n$.

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(5+9)

66

PH 563.1

Reg. No. | | | | | | |

St Aloysius College (Autonomous)
Mangaluru
Semester I– P.G. Examination – M.Sc. Mathematics
November - 2017
REAL ANALYSIS - I

Time: 3 hrs. **Max Marks: 70**

Answer any FIVE full questions **(14x5=70)**

1. a) Define the notion of supremum for a bounded subset E of an ordered set S.
   b) If z and w are complex numbers then show that $||z| - |w|| \leq |z - w|$.
   c) State and prove the Archimedian Property of $\mathbb{R}$.
   d) State and prove Schwarz inequality. **(2+2+5+5)**
2. a) Define a countable set. Prove that every infinite subset of a countable set is countable.
   b) Let A be a countable set and let B_n be the set of all n-tuples of elements of A. Prove that B_n is countable.
   c) Verify whether $d(x, y) = (x - y)^2, x, y \in \mathbb{R}$ is a metric on $\mathbb{R}$.
   d) In a metric space prove that any neighbourhood of a point is an open set. **(4+5+2+3)**
3. a) In a metric space, prove that finite intersection of open sets is open.
   b) Prove that every compact subset of a metric space is closed.
   c) If $\{K_\alpha\}$ is a collection of compact subsets of a metric space X such that the intersection of every finite subcollection of $\{K_\alpha\}$ is non empty, then prove that $\cap K_\alpha$ is nonempty. **(2+6+6)**
4. a) Show that every nonempty perfect set in $\mathbb{R}^k$ is uncountable.
   b) Prove that a subset E of $\mathbb{R}$ is connected if and only if it has the following property: if $x, y \in E$ and $x < z < y$, then $z \in E$. **(7+7)**
5. a) In a metric space prove that every convergent sequence is a Cauchy sequence.
   b) If X is a compact metric space and if $\{p_n\}$ is a Cauchy sequence in X, then show that $\{p_n\}$ converges in X.
   c) Prove that a monotonic increasing sequence of real numbers is convergent if and only if it is bounded. **(2+7+5)**
6. a) Prove that $\lim_{n\to\infty} \frac{n^\alpha}{(1+p)^n} = 0, p > 0$ *and* $\alpha \in \mathbb{R}$.
   b) If $|a_n| \leq c_n$ for $n \geq n_o$ where n_o is some fixed integer and if $\sum c_n$ converges than prove that $\sum a_n$ converges.
   c) Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$
   d) Prove that 'e' is irrational. **(3+4+2+5)**
7. a) Let $f: X \to \mathbb{R}$ be a continuous function. Prove that the set $\{p \epsilon X: f(p) = 0\}$ is closed in X.
   b) Prove that any continuous mapping of a compact metric space X into a metric space Y is uniformly continuous.
   c) If $f: (a, b) \to \mathbb{R}$ is a monotonic function, then prove that f has at most countable number of discontinuities. **(2+7+5)**

Contd ...2

(67)

8. a) Prove that a differentiable function is continuous.

b) Suppose the f is continuous on $[a,b]$, f' exists at some point $x \in [a,b]$, g is defined on an interval I which contains the range of f and g is differentiable at the point $f(x)$. If $h(t) = g(f(t)), a \leq t \leq b$, then prove that h is differentiable at x and

$$h'(x) = g'(f(x))f'(x).$$

c) If f is a real differentiable function on $[a,b]$ and $f'(a) < \lambda < f'(b)$, then prove that there is a point $x \in (a,b)$ such that $f'(x) = \lambda$.

d) Examine the differentiability of the function f at $x = 0$ for the function

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$$

(2+5+5+2)

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PS 564.1

Reg. No.

St Aloysius College (Autonomous)
Mangaluru
Semester I– P.G. Examination – M.Sc. Mathematics
November - 2017
GRAPH THEORY

Time: 3 hrs. Max Marks: 70

Answer any FIVE full questions. (14x5=70)

1. Define the intersection number $\omega(G)$ of a graph G. If G is a connected (p,q) graph, with $p \geq 3$, Prove that $\omega(G) = q$ if and only if G has no triangles. (14)

2. a) Show that the maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4}\right]$.

b) For any graph G with 6 points, show that either G or $\overline{G}$ contains a triangle. (10+4)

3. a) If G is a connected graph, with at least 3 points, prove that G is a block if and only if every two points of G lie on a common cycle.

b) Prove that every nontrivial connected graph G has at least two points which are not cut points. (9+5)

4. a) For a (p,q) graph G, Prove that the following are equivalent:

   i) G is a tree

   ii) every two points of G are joined by a unique path.

   iii) G is connected and $p = q + 1$.

   iv) G is acyclic and $p = q + 1$.

b) Prove that every tree has a center consisting of either one point or two adjacent points. (8+6)

5. a) Define the point connectivity $\kappa(G)$ and line-connectivity $\lambda(G)$ for a graph G. Show that $\kappa(G) \leq \lambda(G) \leq \delta(G)$, where $\delta(G)$ denotes the minimum degree of G.

b) If G is a graph with at least $2n$ points such that for any two disjoint sets V_1 and V_2 of n points each there exist n disjoint paths joining V_1 and V_2, then prove that G is n- connected. (9+5)

6. Prove that in a connected graph G, the minimum number of points separating two non adjacent points s and t is the maximum number of disjoint $s-t$ paths. (14)

7. a) Prove that the following statements are equivalent for a connected graph G:

   i) G is Eulerian.

   ii) every point of G has even degree.

   iii) the set of lines of G can be partitioned into cycles.

Contd...2

b) For a plane map G with p vertices, q edges and r faces, prove that $p-q+r=2$.

Deduce the following:

i) If G is a (p,q) plane map in which every face is an n- cycle, then $q=\frac{n(p-2)}{n-2}$.

ii) The graphs K_5 and $K_{3,3}$ are non-planar. (6+8)

8. a) For every (p,q) graph G, Prove that $\frac{p}{\beta_o} \le \chi \le p-\beta_o+1$, where β_o is the point independence number of G.

b) Prove that every planar graph is 5- colorable. (5+9)

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70

PS 565.1 Reg. No: [| | | | | |]

St Aloysius College (Autonomous)

Mangaluru

Semester I – P.G. Examination - M. Sc. Mathematics

November - 2017

FLUID MECHANICS

Time: 3 Hours **Max. Marks: 70**

Answer any FIVE FULL questions from the following **(14x5=70)**

1. a) Define : i) steady and unsteady flow

   ii) uniform and non-uniform flow

   iii) Laminar and Turbulent flow

   b) Show that the equation of continuity reduces to Laplace's equation when the fluid is incompressible and irrotational.

(6+8)

2. a) Describe the Lagrange's and Eulerian methods of describing the fluid flows and distinguish between them.

   b) Define path line and stream line. Determine the stream line and the path of the particles if $u=\frac{x}{1+t}, v=\frac{y}{1+t}, w=\frac{z}{1+t}$.

   c) Establish the relation $\Omega = 2\omega$ connecting the angular velocity ω and the vorticity vector Ω.

(2+6+6)

3. a) State and prove Bernoulli's theorem.

   b) A long pipe is of length l and has slowly tapering cross section. It is inclined at an angle θ to the horizontal and water flows steadily through it from the upper to the lower end. The section at the upper end has twice the radius of the lower end. At the lower end, the water is at atmospheric pressure. If the pressure at the upper end is twice atmospheric, find the exit speed of water.

(7+7)

4. a) Define velocity potential. Show that fluid motion is irrotational if and only if the velocity potential ϕ exists.

   b) Define stream function. Prove that stream function is constant along stream line.

   c) Find the stream function ψ for the given velocity potential $\phi = cx$, where c is a constant.

   d) Show that for two dimensional irrotational motion both ϕ and ψ satisfy Laplace's equation, also prove that the lines ϕ =constant and ψ =constant intersects orthogonally.

(3+3+2+6)

5. a) State and prove Kelvin's circulation theorem. Use it to show that a motion of a fluid is circulation preserving if and only if

$$\frac{\partial \Omega}{\partial t} + \nabla \times (\Omega \times q) = 0.$$

b) State and prove Milne-Thomson circle theorem.

(8+6)

6. a) Find the complex potential for a two dimensional source and a doublet.

b) State and prove Blasius theorem.

(8+6)

7. a) Derive Navier-Stoke's equation.

b) Obtain the velocity distribution for plane-Couette flow.

(8+6)

8. a) Derive the expression for velocity distribution for the flow through a circular pipe.

b) Describe the diffusion of vorticity.

(8+6)

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